Synthetic Division

Divide $f(x) = 2x^3 + 9x^2 + 14x + 5$ by $x - 3$

$$2x^2 + 15x + 59 + \frac{182}{x-3}$$

$$x - 3 \overline{\smash{)}\,2x^3 + 9x^2 + 14x + 5}$$

$$\cancel{-2x^3} + 6x^2$$

$$\cancel{15x^2} + 14x$$

$$\cancel{-15x^2} + 45x$$

$$\cancel{59x} + 5$$

$$\cancel{-59x} + 177$$

$$182$$

Evaluate $f(x) = 2x^3 + 9x^2 + 14x + 5$ when $x = 3$

$$\begin{array}{c|cccc} 3 & 2 & 9 & 14 & 5 \\ & \downarrow & 6 & 45 & 177 \\ \hline & 2 & 15 & 59 & 182 \end{array}$$

$$2x^2 + 15x + 59 + \frac{182}{x-3}$$

Divide using synthetic division:

$(2x^2 - 7x + 10) \div (x - 5)$ put a "5" on the outside

$$\begin{array}{c|ccc} 5 & 2 & -7 & 10 \\ & \downarrow & 10 & 15 \\ \hline & 2 & 3 & 25 \end{array}$$

$$2x + 3 + \frac{25}{x-5}$$

Zero Product Property

if $a \cdot b = 0$
then
$a = 0$
or
$b = 0$

$0 = x^2 + 7x + 12$

$0 = (x+3)(x+4)$

$x + 3 = 0$ or $x + 4 = 0$

$x = -3$ $\quad$ $x = -4$

$0 = (x - 4)(2x + 5)(3x - 7)(x + 5)$

$x - 4 = 0$	$2x + 5 = 0$	$3x - 7 = 0$	$x + 5 = 0$
$x = 4$	$2x = -5$	$3x = 7$	$x = -5$
	$x = -\frac{5}{2}$	$x = \frac{7}{3}$	

Factor Theorem:

If f(k) = 0, [if the remainder is 0]
then x - k is a factor.

Ex. 1. Factor $f(x) = 2x^3 - 11x^2 + 3x + 36$

given that x - 3 is a factor

$$\begin{array}{r|rrrr} 3 & 2 & -11 & 3 & 36 \\ & \downarrow & 6 & -15 & -36 \\ \hline & 2 & -5 & -12 & 0 \end{array}$$

$2x^2 - 5x - 12$

$2x^3 - 11x^2 + 3x + 36 = (x-3)(2x^2 - 5x - 12)$

$(2x+3)(x-4)$

Solutions:

$x - 3 = 0$ $\quad$ $\boxed{x = 3}$

$2x + 3 = 0$ $\quad$ $2x = -3$ $\quad$ $\boxed{x = -\frac{3}{2}}$

$x - 4 = 0$ $\quad$ $\boxed{x = 4}$